Translation:Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Section II

1 Electric phenomena in ponderable bodies that are moving with constant velocity through the stationary aether.

1.1 Transformation of the fundamental equations.

§ 19. From now on it will be assumed that the bodies to be considered are moving at a steady velocity of translation $\mathfrak{p}$, under which we will have to understand in almost all applications, the speed of the earth in its motion around the sun. It would be interesting at first to further develop the theory for stationary bodies, but for brevity's sake let us immediately turn to the more general case. Besides, it may be still set $\mathfrak{p} = 0$.

The treatment of the problems that are now coming into play is most simple, when instead of the co-ordinate system used above, we introduce another one which is rigidly connected with ponderable matter and therefore shares its displacement.

While the coordinates of a point with respect to the fixed system were called *x, y, z*, let those, which refer to the moving system and which I call the *relative* coordinates, denoted by *(x), (y), (z)* for the time being. Until now, all the variable parameters were seen as functions of *x, y, z, t*; furthermore $\mathfrak{d}_x$, $\mathfrak{d}_y$, etc. shall be seen as functions of *(x) (y), (z)* and *t.*

Under a *fixed* point, we now understand one point, that has a steady position with respect to the new axis; in the same way, by *rest* or *motion* of a physical particle we shall mean the *relative* rest or the *relative* motion in relation to ponderable matter. With ions, which move in this sense of the word, we will have to do as soon as the displaced matter is the seat of electric motions.

By $\mathfrak{v}$ we shall not represent the real velocity, but the velocity of the previously mentioned relative motion. The real velocity is thus

$$\mathfrak{p} + \mathfrak{v},$$

and hereby $\mathfrak{v}$ is to be replaced in equations (4) and (V).

In addition, we have, instead of the derivatives with respect to *x, y, z* and *t*, to establish such with respect to *(x), (y), (z)* and *t.*

The first mentioned derivative I denote by

$$\frac{\partial}{\partial x},\ \frac{\partial}{\partial y},\ \frac{\partial}{\partial z},\ \left(\frac{\partial}{\partial t}\right)_1,$$

however, the latter by

$$\frac{\partial}{\partial (x)},\ \frac{\partial}{\partial (y)},\ \frac{\partial}{\partial (z)},\ \left(\frac{\partial}{\partial t}\right)_2.$$

Now we have, by application to an arbitrary function,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial (x)},\ \frac{\partial}{\partial y} = \frac{\partial}{\partial (y)},\ \frac{\partial}{\partial z} = \frac{\partial}{\partial (z)},$$
$$\left(\frac{\partial}{\partial t}\right)_1 = \left(\frac{\partial}{\partial t}\right)_2 - \mathfrak{p}_x\frac{\partial}{\partial (x)} - \mathfrak{p}_y\frac{\partial}{\partial (y)} - \mathfrak{p}_z\frac{\partial}{\partial (z)}.$$

By that it follows, that we can write for $Div\ \mathfrak{A}$ the expression

$$\frac{\partial \mathfrak{A}_x}{\partial (x)} + \frac{\partial \mathfrak{A}_y}{\partial (y)} + \frac{\partial \mathfrak{A}_z}{\partial (z)},$$

and for the components of $Rot\ \mathfrak{A}$

$$\frac{\partial \mathfrak{A}_z}{\partial (y)} - \frac{\partial \mathfrak{A}_y}{\partial (z)}\text{etc.},$$

The expressions $Div\ \mathfrak{A}$ and $Rot\ \mathfrak{A}$ have still the meaning given in § 4, *g* and *h*, if, after having abandoned the old coordinates one and for all, for simplification we don't indicate the new ones with *(x), (y), (z)*, but with *x, y, z.*

We also want, after we have passed to the new coordinates, use the sign $\frac{\partial}{\partial t}$ instead of $\left(\frac{\partial}{\partial t}\right)_2$ for a differentiation with respect to time at constant relative coordinates, so that

The derivative with respect to time, which occurs in the basic equations (I) - (V), are all of the kind indicated by $\left(\frac{\partial}{\partial t}\right)_1$. We will maintain this sign as an abbreviation for the longer term (18).

In contrast, a point over a letter shall henceforth — such as $\partial/\partial t$ - indicate a differentiation with respect to time at constant relative coordinates. Thus the terms $\dot{\mathfrak{d}}$ and $\dot{\mathfrak{H}}$ in (4) and (IV) may not be left unaltered. By $\mathfrak{d}$, for example, we understood a vector with components

$$\left(\frac{\partial \mathfrak{d}_x}{\partial t}\right)_1 \text{etc.},,$$

or

$$\left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} - \mathfrak{p}_y \frac{\partial}{\partial y} - \mathfrak{p}_z \frac{\partial}{\partial z}\right) \mathfrak{d}_x \text{etc.},$$

We can suitably write this vector

$$\left(\frac{\partial \mathfrak{d}}{\partial t}\right)_1,$$

while

$$\dot{\mathfrak{d}} \text{ or } \frac{\partial \mathfrak{d}}{\partial t}$$

will mean the vector with components

$$\frac{\partial \mathfrak{d}_x}{\partial t} \text{etc.},$$

Based on the system of axes associated with ponderable matter, eventually the fundamental equations become

§ 20. For some purposes, a different form of some equations is more appropriate.

The first of the three (IV) summarized relations is namely

$$-4\pi V^2 \left(\frac{\partial \mathfrak{d}_z}{\partial y} - \frac{\partial \mathfrak{d}_y}{\partial z}\right) = \frac{\partial \mathfrak{H}_x}{\partial t} - \mathfrak{p}_x \frac{\partial \mathfrak{H}_x}{\partial x} - \mathfrak{p}_y \frac{\partial \mathfrak{H}_x}{\partial y} - \mathfrak{p}_z \frac{\partial \mathfrak{H}_x}{\partial z},$$

where, by equation (II_a), we can write for the last three members

$$\left(\mathfrak{p}_x \frac{\partial \mathfrak{H}_y}{\partial y} - \mathfrak{p}_y \frac{\partial \mathfrak{H}_x}{\partial y}\right) - \left(\mathfrak{p}_z \frac{\partial \mathfrak{H}_x}{\partial z} - \mathfrak{p}_x \frac{\partial \mathfrak{H}_z}{\partial z}\right),$$

which is nothing else than the first component of

$$Rot\ [\mathfrak{p}.\mathfrak{H}].$$

Accordingly, we obtain instead of (IV_a)

$$Rot\ \left\{4\pi V^2 \mathfrak{d} + [\mathfrak{p}.\mathfrak{H}]\right\} = -\dot{\mathfrak{H}}.$$

Furthermore, the current $\mathfrak{S}$ can be entirely eliminated. The first of equations (III_a) becomes, when we consider (4_a) and (I_a),

$$\frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} = 4\pi\rho\left(\mathfrak{p}_x + \mathfrak{v}_x\right) + 4\pi\left(\frac{\partial \mathfrak{d}_x}{\partial t} - \mathfrak{p}_x \frac{\partial \mathfrak{d}_x}{\partial x} - \mathfrak{p}_y \frac{\partial \mathfrak{d}_x}{\partial y} - \mathfrak{p}_z \frac{\partial \mathfrak{d}_x}{\partial z}\right) = 4\pi\rho\mathfrak{v}_x + 4\pi\left\{\left(\mathfrak{p}_x \frac{\partial \mathfrak{d}_y}{\partial y} - \mathfrak{p}_y \frac{\partial \mathfrak{d}_x}{\partial y}\right) - \left(\mathfrak{p}_z \frac{\partial \mathfrak{d}_x}{\partial z} - \mathfrak{p}_x \frac{\partial \mathfrak{d}_z}{\partial z}\right)\right\} + 4\pi \frac{\partial \mathfrak{d}_x}{\partial t}.$$

By that it follows, if we define a new vector $\mathfrak{H}'$ by means of the equation

$$\mathfrak{H}' = \mathfrak{H} - 4\pi[\mathfrak{p}.\mathfrak{d}],$$

thus

$$Rot\ \mathfrak{H}' = 4\pi\rho\mathfrak{v} + 4\pi\dot{\mathfrak{d}}.$$

If we now introduce the sign $\mathfrak{F}$ for the electric force-action on *stationary* ions, we obtain the following set of formulas

§ 21. From equations (I_a)—(V_a) (§ 19) also some formulas can be derived, any of them contains only one of the magnitudes $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$, $\mathfrak{H}_x$, $\mathfrak{H}_y$, $\mathfrak{H}_z$.

At first, it follows from (IV_a)

$$-4\pi V^2 Rot\ Rot\ \mathfrak{d} = Rot\left(\frac{\partial \mathfrak{H}}{\partial t}\right)_1 = \left(\frac{\partial Rot\ \mathfrak{H}}{\partial t}\right)_1.$$

If we consider here what has been said in § 4, *h*, as well as the relations (I_a), (III_a) and (4_a), we arrive at the three formulas

Similarly, we find

$$V^2 \Delta \mathfrak{H}_x - \left(\frac{\partial^2 \mathfrak{H}_x}{\partial t^2}\right)_1 = 4\pi V^2 \left[\frac{\partial}{\partial z}\left\{\rho\left(\mathfrak{p}_y + \mathfrak{v}_y\right)\right\} -\right.$$

The last members of these six equations are completely known once we know how the ions are moving.

1.2 Application to electrostatics.

§ 22. We want to calculate by which forces the ions act on one another, when all of them are at rest with respect to ponderable matter. In this case a state occurs, where at every point $\mathfrak{d}$ and $\mathfrak{H}$ are independent of time. We have

and equations (A) and (B) will be reduced, when for brevity's sake the operation

$$\Delta - \frac{1}{V^2}\left(\mathfrak{p}_x \frac{\partial}{\partial x} + \mathfrak{p}_y \frac{\partial}{\partial y} + \mathfrak{p}_z \frac{\partial}{\partial z}\right)^2$$

is indicated by Δ', to

and

To fulfill these conditions, we determine a function ω by

$$\Delta'\omega = \rho$$

and put

i.e., values that really satisfy the fundamental equations (I_a) - (IV_a).

From (V_a) it also follows

so that the sought forces are found.

Without prejudice to generality, we may assume that the translation happens in the direction of the *x*-axis. It is then $\mathfrak{p}_y = \mathfrak{p}_z = 0$, and the formula for the determination of ω will be transformed into

§ 23. To clearly define the meaning of the above formulas, we will compare the considered system S_1 with a second one S_2 . The latter should *not* be moved, and it arises from S_1 by increasing all the dimensions that have the direction of the *x*-axis (therefore the relevant dimensions of the ions as well), in the ratio $\sqrt{V^2 - \mathfrak{p}^2}$ to V , or: between the coordinates *x, y, z* of a point of S_1 and the coordinates x', y', z' of the same corresponding point of S_2 , we let remain the relations

In addition, the mutually corresponding volume elements, and therefore also the ions, shall have the same charges in S_1 and S_2 .

If we apply to all magnitudes, which are related to the second system, a prime so they can be distinguished, then

$$\rho' = \rho\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}},$$

and

$$\frac{\partial^2\omega'}{\partial x'^2} + \frac{\partial^2\omega'}{\partial y'^2} + \frac{\partial^2\omega'}{\partial z'^2} = \rho' = \rho\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}.$$

Then the equation (23) can be written in the form

$$\frac{\partial^2\omega}{\partial x'^2} + \frac{\partial^2\omega}{\partial y'^2} + \frac{\partial^2\omega}{\partial z'^2} = \rho$$

then

$$\omega = \frac{\omega'}{\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}},$$

and since in the second system

$$\mathfrak{E}'_x = 4\pi V^2 \frac{\partial\omega'}{\partial x'} \text{etc.},$$

thus

$$\mathfrak{E}_x = \mathfrak{E}'_x,\ \mathfrak{E}_y = \sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}\mathfrak{E}'_y,\ \mathfrak{E}_z = \sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}\mathfrak{E}'_z.$$

The same relations, as they exist between the components of $\mathfrak{E}$ and $\mathfrak{E}'$, also exist, since the charges in S_1 and S_2 are equal, between the force components acting on an ion.

If in the second system at certain places $\mathfrak{E}' = 0$, then $\mathfrak{E}$ vanishes at the corresponding points of the first system.

§ 24. Several implications of this theorem are obvious. From ordinary electrostatics, we know for example that an excess of positive (or negative) ions can be distributed over a conductor, namely over its surface Σ , so that in the interior no electric force is acting. If we take this distribution for the system S_2 and derive from it a system S_1 by the above-discussed transformation, then also in this one an excess of positive ions only exists at a certain surface Σ , while in all interior points the electric force $\mathfrak{E}$ vanishes. The fact that an electric charge is located at the *surface* of a conductor, won't be changed by the translation of ponderable matter.

Similar considerations apply to two or more bodies. If a conductor *C* is confronted with a charged body *K*, then there exists, according to a known theorem, always a certain amount of charge on the surface of *C*, which together with *K* exerts no action on the ions in the interior of the conductor. This theorem remains valid, if the ponderable matter is moving, and it is even still allowed to assume that, under the influence of *K*, an "induced" charge is formed by itself upon *C*, which just cancels the effect of *K* on the interior points.

Since by (22) the components of $\mathfrak{E}$ are proportional to the derivative of ω, we can also say that inducing and inducted charges together cause a constant ω at all points of *C*. It follows then by means of equations (20), (21) and (V_a), that also a moving ion in the interior of C does not experience any force-action from the two charges.

Finally, it should be noted that by our formulas, the distribution of a charge over a given conductor, as well as the attraction or repulsion of charged bodies by the motion of the earth, must be changed. But this influence is limited to the *second* order, namely if the fraction $\mathfrak{p}/V$ is called a magnitude of *first* order, and thus the fraction $\mathfrak{p}^2/V^2$ is called a magnitude of *second* order.

Since $\mathfrak{p}/V = 1/10000$, we may not hope, neglecting some very special cases, to find with respect to electrical and optical phenomena an influence of earth's motion that depends on $\mathfrak{p}^2/V^2$. The only thing that could be observed in relation to bodies at rest on earth, is the magnetic force (21). At first glance, we might expect a corresponding effect on the current elements. We will return to this question in § 26.

1.3 Values of $\mathfrak{d}$ and $\mathfrak{H}$ at a stationary current.

§ 25. On the basis of equations (A) and (B) we again tackle the problem treated in § 11. We consider, as there, the mean values and take into account that for them the simplification (19) is permitted in stationary states; moreover, we assume at first that the conductors do not have a significant charge, so that $\bar{\rho} = 0$.

It is near at hand to interpret the vector $\overline{\rho\mathfrak{v}}$ as being a "current". We think of it as solenoidally distributed and denote it by $\bar{\mathfrak{S}}$, where it remains, however, temporarily undecided whether this is also the mean value of the vector occurring in (4_a).

We now derive from (A) and (B)

$$V^2\Delta'\bar{\mathfrak{d}}_x = -\left(\mathfrak{p}_x\frac{\partial}{\partial x} + \mathfrak{p}_y\frac{\partial}{\partial y} + \mathfrak{p}_z\frac{\partial}{\partial z}\right)\bar{\mathfrak{S}}_x \text{etc.},,$$

$$\Delta'\overline{\mathfrak{H}_x} = 4\pi\left(\frac{\partial\overline{\mathfrak{S}}_y}{\partial z} - \frac{\partial\overline{\mathfrak{S}}_z}{\partial y}\right) \text{ etc. },$$

If we determine thus the three auxiliary magnitudes χ_x , χ_y , χ_z [1] by means of the equations

$$\Delta'\chi_x = \overline{\mathfrak{S}}_x,\ \Delta'\chi_y = \overline{\mathfrak{S}}_y,\ \Delta'\chi_z = \overline{\mathfrak{S}}_z,$$

so everywhere we have

and by (V_a) the electric force acting on stationary ions,

At first glance, it therefore seems as if a current that streams through a conductor, is acting on a stationary ion with a force of first order. However, on closer reflection we find that the force (27) is just being compensated by another force.

The values (27) are in fact in perfect agreement with the expressions (22), if we substitute

By § 22, ω would belong to an electric charge, its density is

$$\rho = \Delta'\omega,$$

or by the given formulas

Let us imagine for a moment that the current does not exist, but there is a charge with the average density ρ. This would of course exist only in the conductor, and the total sum would be zero, as it follows from (29) and

$$\Delta\overline{\mathfrak{S}} = 0$$

Obviously this ion distribution would completely vanish, if it is left alone. This can also be expressed by saying that the charge will set them in motion by virtue of its action on resting ions, and that therefore eventually another charge with the average density -ρ occurs besides it, or

$$\frac{\mathfrak{p}_x\overline{\mathfrak{S}_x}+\mathfrak{p}_y\overline{\mathfrak{S}_y}+\mathfrak{p}_z\overline{\mathfrak{S}_z}}{V^2-\mathfrak{p}^2}$$

Since the current that we considered initially, exactly acts on resting ions as the charge (29), it will also generate the charge *A* after a short time; this eliminates the effects on stationary ions, namely not only in the outer points, but also, at least with respect to the averages of the forces, in the interior of the conductor.

I want to call this charge A the *compensation charge*. Once generated, the conductor does not cause any electricity motion in a neighboring body. A stationary current in a wire moving with the Earth therefore exerts no inductive action on a circuit which is also at rest with respect to Earth, regardless of Earth's motion[2].

It should be noted now that in the finally occurring state of the system, ρ and $\mathfrak{d}$ have certain values of order $\mathfrak{p}$. Neglecting the magnitudes of second order, then it really follows from (4_a)

$$\overline{\mathfrak{S}} = \overline{\rho\mathfrak{v}}.$$

1.4 Interaction between a charged body *K* and a conductor.

§ 26. After the foregoing, we have to assume that in the conductor next to the current $\overline{\mathfrak{S}}$, a compensation charge does exist, and also (at the surface of the conductor) the electrostatic induction-charge *B* caused by *K*. For simplicity, we imagine that $\overline{\mathfrak{S}}$, A and B co-exist as independent ion systems[3]. Each of the four systems $\overline{\mathfrak{S}}$, A , B and K now forces a special state to the aether, and thus acts on any of the others. To shortly indicate these actions, we want to put $\left(\overline{\mathfrak{S}}, K\right)$ for those actions, which for example were exerted by $\overline{\mathfrak{S}}$ on K , where we have to notice that perhaps $\left(\overline{\mathfrak{S}}, K\right)$ and $\left(K, \overline{\mathfrak{S}}\right)$ are not equal and opposite, and that also actions such as $\left(\overline{\mathfrak{S}}, \overline{\mathfrak{S}}\right)$ may exist, namely forces which act on one of the ion systems due to condition changes in the aether, which were caused by itself.

In easily understandable symbols we can now write for the total action on *K*

$$(K,K) + (B,K) + (\overline{\mathfrak{S}},K) + (A,K),$$

which, however, due to § 25

$$(\overline{\mathfrak{S}},K) + (A,K) = 0$$

is reduced to the first two terms and thus becomes independent of the current.

On the other hand, the forces which act on the conductor, can be represented by a expression consisting of 12 members, since the action of K , $\overline{\mathfrak{S}}$, A and B on $\overline{\mathfrak{S}}$, A and B , has to be considered each time. It is now

$$(K,\overline{\mathfrak{S}}) + (B,\overline{\mathfrak{S}}) = 0,\ (K,A) + (B,A) = 0,$$

so that by the aforementioned expression it only remains

Those forces represented by the first two members would also exist, when $\overline{\mathfrak{S}} = 0$, and the last two members are independent of the charged body *K*. An action of *K* exerted on the conductor as such, doesn't exist.

Besides, in each of the four members (30), the part that depends on $\mathfrak{p}$ is of second order. We already know this from $(K,B) + (B,B)$, since this represents an electrostatic effect. $(A,\overline{\mathfrak{S}})$ and $(\overline{\mathfrak{S}},\overline{\mathfrak{S}})$, however, represent forces acting on a current, in which the mean electric density is zero. As it can be seen from (V_a), such forces are determined by the value of $\mathfrak{H}$, which belongs to the *acting* system. Inasmuch as $\mathfrak{H}$ (that belongs to $\overline{\mathfrak{S}}$) depends on $\mathfrak{p}$, it is of second order (§ 25), and the compensation charge A only produces by its velocity $\mathfrak{p}$ a magnetic force of second order, since its density already contains the factor $\mathfrak{p}/V$.

1.5 Electrodynamic actions.

§ 27. The question as to how these effects are influenced by earth's motion, can now easily be answered. If we denote the currents in two conductors by $\overline{\mathfrak{S}}$ and $\overline{\mathfrak{S}'}$, and the corresponding compensation charges by *A* and A', then the action exerted on the second conductor is

$$(\overline{\mathfrak{S}}, \overline{\mathfrak{S}'}) + (A, \overline{\mathfrak{S}'}) + (\overline{\mathfrak{S}}, A') + (A, A'),$$

in which the last two terms are mutually canceled. That $(A, \overline{\mathfrak{S}'})$ and the $\mathfrak{p}$ -dependent part $(\overline{\mathfrak{S}}, \overline{\mathfrak{S}'})$ are of order $\mathfrak{p}^2/V^2$, follows from considerations such as those communicated above.

1.6 Induction in a linear conductor.

§ 28. A closed secondary wire from *B* will be displaced from B_1 into position B_2, while a primary conductor *A* at the same time passes from position A_1 to A_2, and the intensity of the primary current increases from i_1 to i_2. At the beginning and the end of time *T*, in which these processes take place, the two conductors shall be at rest and the primary current shall be constant; if no other electromotive forces acts on *B*, then this wire will eventually be, as before, without current. We want to determine the quantity of electricity, which has passed in time *T* through a cross section of the wire, and namely we will *only* consider *the convection current* at this place.

After the expiry of the whole process, the surface of *B* has nowhere a electric charge. It follows that the quantity of electricity that streamed through is the same for all cross sections, and that the conductor can be decomposed into infinitely thin current tubes, so that in each of them and equally through all cross-sections, the same quantity of electricity is streaming.

We consider in detail one of these tubes, and call *ds* an element of their length, ω is a vertical cross-section, *Ndt* the number of positive ions which pass through it during the time *dt* in the assumed positive direction *s*, *N'dt* the number of negative ions which move in the opposite direction, *e* is the charge of a positive and $-e'$ the charge of a negative ion. The total current through ω is then

Furthermore, $\mathfrak{E}_s$ and $\mathfrak{E}'_s$ are the electric forces acting in the direction of *ds*, which come into consideration for a positive or a negative ion. By Ohm's law we shall assume, that the motion of ions by these forces is thus determined, so that N and N' are proportional to its mean value; this and the proportionality to ω, we express by

$$N = p\overline{\mathfrak{E}_s}\omega,\ N' = q\overline{\mathfrak{E}'_s}\omega,$$

where *p* and *q* are constant factors.

It is now necessary to distinguish between the velocity of the considered conductor element and the relative velocity of an ion in the wire. The former shall be called $\mathfrak{v}$ and the latter $\mathfrak{w}$. From (V_a) it is given

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{p}.\mathfrak{H}] + [\mathfrak{v}.\mathfrak{H}] + [\mathfrak{w}.\mathfrak{H}].$$

Yet, the velocity $\mathfrak{w}$ has the direction of *ds*; consequently we have $[\mathfrak{w}.\mathfrak{H}]_s = 0$, and for positive as well as for negative ions

$$\mathfrak{E}_s = \mathfrak{E}'_s = 4\pi V^2\mathfrak{d}_s + [\mathfrak{p}.\mathfrak{H}]_s + [\mathfrak{v}.\mathfrak{H}]_s.$$

Finally, equation (31) transforms into

$$i = c\omega \int \left\{4\pi V^2\bar{\mathfrak{d}}_s + [\mathfrak{p}.\bar{\mathfrak{H}}]_s + [\mathfrak{v}.\bar{\mathfrak{H}}]_s\right\}\ d\,t,$$
$$c = pe + qe'.$$

Let us divide by $c\,\omega$, multiply by *ds*, and integrate over the whole current-line. If we consider here, that *i* has everywhere the same value in the current-line, and if we put

$$\int \frac{d\,s}{c\,\omega} = \frac{1}{C},$$

we shall find

§ 29. The following discussion is intended to derive the known fundamental law of induction from this formula. Imagine an area σ on which the current-line constantly is located during its motion, and consider the integral

for the part that is cut by the line.

This quantity, which is usually called "the number of magnetic force-lines covered by *s*", changes over time, namely for two reasons. First, $\bar{\mathfrak{H}}$ varies at each point, and second, the area of integration changes.

During time *dt*, the first cause produces the following increase of *P*

$$d\,t \int \dot{\overline{\mathfrak{H}_n}}\, d\,\sigma.$$

As to the second variation, it should be noted that each element *ds* describes an infinitely small parallelogram on the surface, and that the value of the surface integral $\int \dot{\overline{\mathfrak{H}_n}}\, d\,\sigma$ of this parallelogram, by suitably chosen signs, goes into *dP*. This value is determined by the area of the parallelepiped, with $d\,s$, $\mathfrak{H}$ as its sides, and the distance $\mathfrak{v}\,d\,t$ in the direction of $\mathfrak{v}$. We will find for it

$$-d\,t[\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s,$$

and for the whole increase of (33)

$$d\,P = d\,t \int \dot{\overline{\mathfrak{H}_n}}\,d\,\sigma - d\,t \int [\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s,$$

or, if the relations (IV_b) and (V_b), as well as the theorem stated in (1) (§ 4, *h*), were considered,

$$-d\,t \int \left\{4\pi V^2 \bar{\mathfrak{d}}_s + [\mathfrak{p}.\bar{\mathfrak{H}}]_s\right\}\ d\,s - d\,t \int [\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s.$$

Consequently, (32) transforms into

$$i = -C \int d\,P = C\,(P_1 + P_2)\,,$$

where P_1 and P_2 belong to the beginning and the end of the considered time.

The magnitude *P* depends on the different parts of $\mathfrak{H}$. Since an induced current neither exists at the beginning nor at the end of time *T*, we commit no mistake when we substitute into (33) for $\mathfrak{H}$ only the magnetic force generated by the primary current. The prime above the letter can be omitted here, and if the induced wire is very thin, we may calculate for all current-lines with the same *P*. Finally, if C_1 is the sum of all numbers *C* (*i.e.*, the conductivity of the induced electrical circuit), then the integral-current which we wished to calculate, becomes

$$I = C_1\,(P_1 - P_2)\,,$$

which is consistent with a known theorem.

The motion of Earth was never overlooked during the given derivation; consequently the formula admits of a conclusion about the influence of this motion on the phenomena of induction. There, only magnitudes of second order come into account. $\mathfrak{H}$, which should serve to determine the magnitude *P*, is indeed composed of the vector specified by (26) and the magnetic force which is generated by the compensation charge. The latter magnetic force is of order $\mathfrak{p}^2/V^2$, and since in equations (§ 25) that serve to determine χ_x , χ_y , χ_z , also just the square of $\mathfrak{p}$ is included, then the values (26) differ only to second order from the expressions that apply to a stationary earth.

By proving, that no first-order influence may be expected from the phenomena of induction, we have achieved the explanation for the negative result of Des Coudres[4].

[1] These magnitudes are only different by a constant factor from the components of the vector potential, when $\mathfrak{p} = 0$.

[2] It should be remembered that Mr. Budde (Wied. Ann., Vol 10, p. 553, 1880), on the basis of Clausius' law, reached the same conclusions, as it was drawn here by me. His value for the density of the compensation current even completely agrees with the above-found, if $\mathfrak{p}^2$ is neglected.

[3] This mode of imagination, however, is in no way necessary. To show that the considerations communicated in the texts are correct, we don't need to assume, that the ions which form the charges A and B, were remaining at rest and were altogether uninfluenced by the adjacent existing current. We can also imagine that *all* ions are moving, similar to an electrolyte, in a most irregular manner. But a constant, non-zero mean value $\bar{ho}$ is very well possible; because this constitutes the charges designated by *A* and *B* (*i.e.*, $\bar{\rho}$ is composed of two terms of a sum $\bar{\rho_A}$ and $\bar{\rho_B}$), while the current $\bar{\mathfrak{S}}$ is determined by $\overline{\rho\mathfrak{v}}$. If in (A) and (B) all members are replaced by the mean values, one easily sees that each of the vectors $\bar{\mathfrak{d}}$ and $\bar{\mathfrak{H}}$ consists of two parts, where one of them only depends on $\bar{\rho}$ and the other one only depends on $\overline{\rho\mathfrak{v}}$. Now, as the actions to the outside were determined by those vectors, then they are just so, as if the charge and the current were not connected with each other at all. The same is true for the actions *exerted* on the conductor. Namely, if $\mathfrak{d}$ and $\mathfrak{H}$ are the variations caused by external causes in the aether, then by (V_a) the force acting on a volume element is given by

$$4\pi V^2 \rho\mathfrak{d}\;d\,\tau + \rho[\mathfrak{p}.\mathfrak{H}]\;d\,\tau + \rho[\mathfrak{v}.\mathfrak{H}]\;d\,\tau.$$

The action, to which a noticeable part of the body is subjected, can thus be calculated in a manner, by which we put as unit volume

$$4\pi V^2 \bar{\rho}\mathfrak{d} + \bar{\rho}[\mathfrak{p}.\mathfrak{H}] + [\bar{\rho}\mathfrak{v}.\mathfrak{H}],$$

which again decomposes into two parts $\bar{\rho}$ and $\overline{\rho\mathfrak{v}}$. Strictly taken, also a *third* charge would have to be taken into account. The current can not exist without a potential gradient, and this cannot exist without electric charges of the parts of the conductor. These charges, however, play in the considered questions no essential role, and could even more be left out, as we can think of them as vanishingly small if we assume a very high conductivity.

[4] Actually, we would have to consider now, under consideration of the Earth's motion, the effect of the induction of a galvanometer. In the experiments of Des Coudres (Wied. Ann., Vol 88, p. 71, 1889) an induction role was located between two successive connected primary roles, which have been streamed by the current, so that its effects are just compensated. Since, whatever influence the translation may have by the way, the galvanometer must remain at rest if *I* disappears, thus we may infer from the theory that, neglecting magnitudes of second order, the compensation is not disturbed by Earth's motion.

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2.1 Text

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